EEC 4230 - Mobile Communication Systems

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Lecture 4: Mobile radio propagation: Large-scale path loss-I

Outline

Quiz

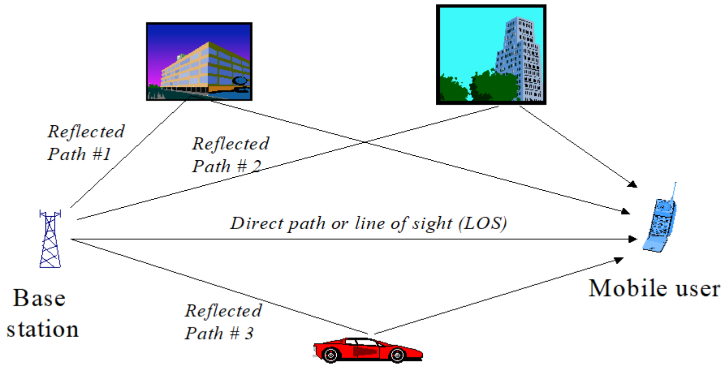
Quiz 1

The SIR received by a mobile at the cell edge in a cellular system can be roughly approximated as: $S/I = \frac{R^{-n}}{i_o D^{-n}}$, where D is the distance between the serving BS and the co-channel cells i_0, and R is the radius of a cell. Find the appropriate cluster size to be used for a GSM network operating in an urban area with path loss exponent of 4 (consider only the first tier of co-channel BSs, and assume that the required SIR for GSM is 41 dBm in order to keep BER $< 10^{-3}$)

Introduction

Unlike wired channels that are stationary and predictable, mobile radio channels are extremely random and not easily analyzed or predicted.

How do we predict these random propagation channel effects on the transmitted signals in mobile communication systems?. [That is needed for planning & designs]



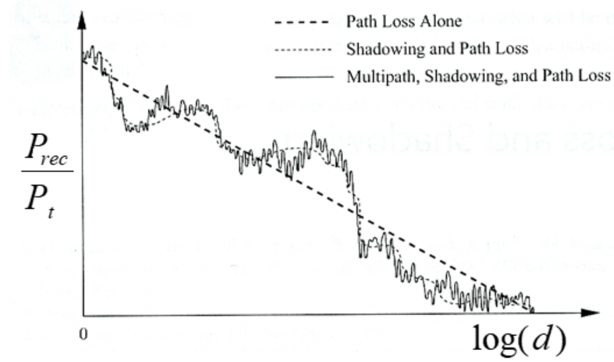
It is done in statistical fashion based on real measurements for each individual communications system or frequency spectrum.

Propagation models

Propagation models have been developed for predicting the average received signal strength (RSS) at a given distance from the TX, as well as the variations in the RSS.

Propagation Models

- Large-scale models: for predicting the RSS as a function of distance from the BS and it include: (1) Deterministic (2) Shadowing (3) Empirical models
- Small-scale fading models: for predicting random variations in the average RSS such as statistical models.



Small-scale fading are sometimes called multipath fading because contribution of multipath to this fading type is huge.

Propagation models

Large scale propagation models

- To predict the mean signal strength over large transmitter-receiver distances (thousands of meters)(or 5 to 40 wavelength).
- It is useful in estimating radio coverage area of TX (cell) as well as interference among cells.
- Factors: (1) Free space propagation loss (path loss), (2) Ground reflections, (3) Diffractions and scattering, (4) Shadowing (caused by mobile environment).

Small scale propagation models:

- To characterize the rapid fluctuations of the received signal strength over very short distances (a few wavelengths) or over short time durations (seconds).
- Factors: (1) Multipath propagation, (2). Speed of the mobile, (3). Speed of the surrounding objects, (4). Bandwidth of the transmitted signals.

Basic Mechanisms Impacting Radio Propagation

Three basic mechanisms impacting both Large scale & Small scale propagations:

(1) Reflection

arise when a propagating electromagnetic wave falls on objects much larger than its wavelength, causing reflections e.g. Earth surface, buildings, walls.

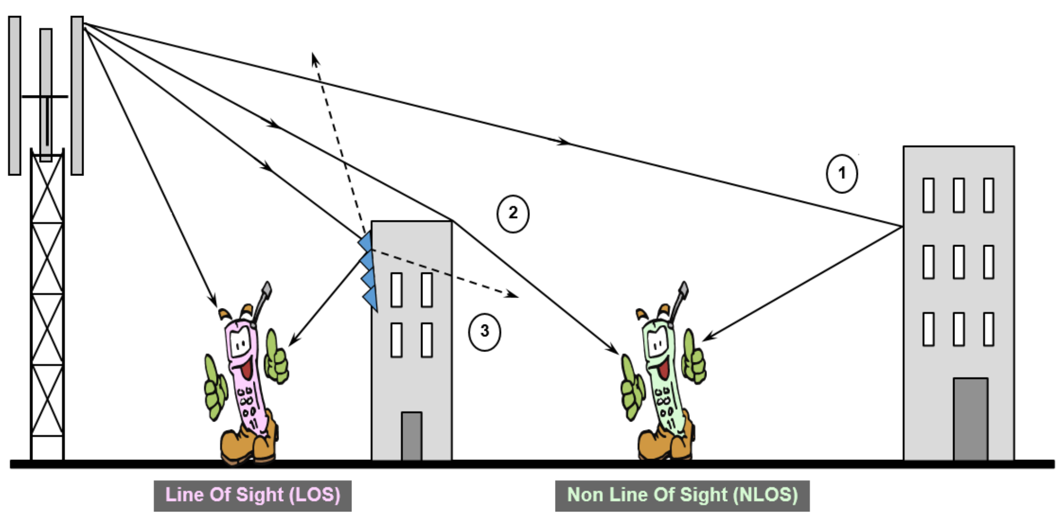
(2) Diffractions

Occurs according to Huygens's principle when there is an obstruction with sharp edges between TX and RX and secondary waves are generated behind the obstructing body. The emergent wave over the obstacle edge is bent at an angle different from the incident angle on the obstacle.

(3) Scattering

Occurs when propagating waves hits objects along its path, whose dimensions are small compared to its wavelength, and when the number of obstacles per unit space is too large, causing the energy to be redirected in many directions (e.g. downtown area).

Basic Mechanisms Impacting Radio Propagation



(1) Free Space Propagation Model

- Used to predict received signal strength when the TX and TX have a clear, unobstructed line-of-sight (LOS) path between them.
- Satellite communication systems and micro-wave LOS radio links typically undergo free space propagations.
- The power received by a receiving antenna separated from the transmitting antenna by a distance d, is given by Frii's equations as:

$$P_r(d) = P_t G_t G_r (\frac{\lambda}{4\pi d})^2 (\frac{1}{L}) \qquad \text{Watts}$$

P_t : Transmitted power
$P_r(d)$: Received power at distance d
G_t : Transmitting antenna gain
G_r : Receiving antenna gain
L : System loss factor, $L \geq 1$
$\lambda = c/f$: Wavelength of the transmitted RF signal.
$L_p = (\frac{4\pi d}{\lambda})^2$: this term is called the free space path loss.

(1) Free Space Propagation Model

Example 1:

If a TX applies 50W of power, to a unity gain antenna with a 900 MHz carrier frequency, (a) find the received power (in dB) at a free space distance of 100m from the antenna, (b) what is P_r(10km)?. Assume unity gain for the receiver antenna.

Solution

An isotropic (ideal) antenna has unity gain (G =1).

$$P_r(d) = \frac{(50)(1)(1)(\frac{3\times10^8}{900\times10^6})^2}{(4\pi)^2 d^2 (1)} \qquad \text{Watts}$$

- (a) $P_r(100\text{ m})[dB] = -54.5$ dB
- (b) $P_r(10\text{ km})[dB] = -94.5$ dB

The effective isotropic radiated power ($EIRP$) of an antenna is given by $EIRP = P_t \times G_t$.

(1) Free Space Propagation Model

Path Loss for Free Space Propagation:

$$PL(d) = \frac{P_t}{P_r} = \frac{(4\pi)^2 d^2 L}{\lambda^2} \qquad \text{Assuming } G_t, G_r = 1$$

Or it can be defined as the difference in dB between the effective transmitted power and the received power as: $PL[dB] = P_t[dB] - P_r[dB]$.

$$PL(d)[dB] = 10\log_{10}\frac{(4\pi)^2 d^2 L}{\lambda^2} = 32.4478 + 20\log_{10}(d_{km}) + 20\log_{10}(f_{MHz})$$

Equation above is not valid when $d = 0$, therefore large-scale propagation models use a close-in reference distance, d_0, as a known received power reference point. The received power at any distance $d \geq d_0$ is then given by:

$$P_r(d) = P_r(d_0)\left(\frac{d}{d_0}\right)^{-2}$$

Where $P_r(d_0)$ is obtained by measurements.

(1) Free Space Propagation Model

d_0 is typically 100 m or 1 km for outdoor systems and 1 m for indoor systems and it should greater than the far field distance such that $d_0 \geq d_f$ where $d_f = \frac{2D^2}{\lambda}$ and D is the antenna diameter.

Path Loss for Free Space Propagation using close-in reference distance d_0:

$$PL(d)[dB] = PL(d_0)[dB] + 20\log_{10}\left(\frac{d}{d_0}\right)$$

where

$$PL(d_0)[dB] = 32.4478 + 20\log_{10}(d_0) + 20\log_{10}(f_{MHz})$$

Received Power in terms of Electric Field:

The received power at distance d, is related to the electric field intensity as.

$$P_r(d) = \frac{|E|^2 G_r \lambda}{480\pi^2} \qquad \text{Watts.}$$

Where $|E|$ is the magnitude of the radiating electric field (in V/m).

(1) Free Space Propagation Model

Example 1:

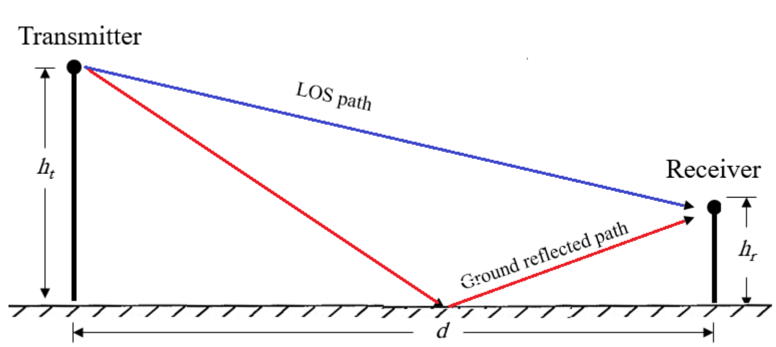
Assume a receiver is located 10km from a 50W transmitter. The carrier frequency is 900MHz, free space propagation is assumed. (a) Find the far-field distance for an antenna with maximum dimension of 1m (b) Assume $G_t = 1$ and $G_r = 2$, find the power at the receiver, (c) find the magnitude of the E-field at the receiver antenna, (d) calculate the rms voltage applied to the receiver input assuming that the receiver antenna has a purely real impedance of 50Ω and is perfectly matched to the receiver (i.e. no loss).

Solution

(a) $d_f = \frac{2D^2}{\lambda} = 6m$
(b) $P_r(d) = -91.5$ dB,
(c) $|E| = 0.0039$ V/m,
(d) $P_r(d) = V^2/R$: thus $V = 0.187$ mV.

(2) Ground Reflections (Two-Ray) Model

- In a mobile radio channel, a single direct path between the BS and a mobile is not always achieved, and hence the free space propagation model is in most cases inaccurate when used alone.
- The 2-ray ground reflected model considers both the direct path and a ground reflected path between TX and RX, and it is found to be reasonably accurate for predicting the large-scale signal strength over distances of several kilometers for mobile radio system with tall towers (over 50m).



(2) Ground Reflections (Two-Ray) Model

The received power at distance d from the transmitter for this model is

$$P_r(d) = \frac{p_t G_t G_r h_t^2 h_r^2}{d^4}$$

The path loss for the 2-ray model is

$$PL(d)[dB] = 40\log_{10} d - 10\log_{10} G_t - 10\log_{10} G_r - 10\log_{10} h_t - 10\log_{10} h_r$$

Example 2:

A mobile is located 5km from a 10W base station transmitter uses an antenna with a gain of 2.55 dB to receive cellular radio signals. Given that the base station antenna is an isotropic radiator and is 50m tall while the mobile station antenna is 1.5 m above the ground, estimate the received power using two-ray model.

Solution:

$P_r(d) = 10\log_{10}\left[P_t G_t G_r h_t^2 h_r^2 / d^4\right] = -98$ dB

(3) Log-Distance Path Loss Model

- Some classical propagation models have emerged overtime, and these are used in practice to predict large-scale coverage for mobile system designs.
- This approach uses the path loss models to estimate the received signal level as a function of distance, and thus predict signal levels at different locations.
- Both theoretical and measurement-based propagation models indicate that average received signal power decrease logarithmically with distance.

Log-Distance Path Loss Model

- Measurements have shown that the average received power, for any mobile communication environment, can be expressed as:

$$\overline{P_r}(d) = \overline{P_r}(d_0)\left(\frac{d}{d_0}\right)^{-n}$$

- n: is the path loss exponent (PLE) which indicates the rate at which the signal power decays with distance for a given environment.

Note that $n = 2$ for the free space model.

(3) Log-Distance Path Loss Model

The average large-scale path loss can thus be expressed as:

$$\overline{PL}(d)[dB] = \overline{PL}(d_0)[dB] + 10n\log_{10}\left(\frac{d}{d_0}\right)$$

Path loss increases with d at a rate proportional to n, or $10n$ dB/decade (similarly for $\overline{P_r}(d)$). Given that we know PL at any location (e.g. by measurements), and we are given the transmitted power, we can estimate the average power as:

$$\overline{P_r}(d)[dB] = P_t[dB] - \overline{PL}(d)[dB]$$

.

The value of n depends on the specific environment

Environment	Path loss exponent "n"
Free space	2
Urban area (cellular radio)	2.7 to 3.5
Shadowed urban cells	3 to 5
In building, line-of-sight	1.6 to 1.8
In building, obstructed path	4 to 6
In factory, obstructed path	2 to 3

d_0 is typically 1 Km for macro-cellular systems and around 100 m for microcellular systems

(4) Log-normal Shadowing Model

- Deterministic models express the path loss as a function of the distance d only.
- It does not take into account the changes of the surrounding environment.
- Statistical models suggest that the path loss at a particular distance, d, is a random variable with Gaussian distribution in dB and mean $\overline{PL}(d)[dB]$:

$$PL(d)[dB] = \overline{PL}(d)[dB] + X_\sigma = \overline{PL}(d_0)[dB] + 10n\log_{10}\left(\frac{d}{d_0}\right) + X_\sigma$$

- X_σ is a zero-mean Gaussian distributed random variable in dB (log-normal) with standard deviation σ, also in dB.
- In practice, the values of PLE n and the path loss variance σ^2 (or standard deviation σ) are computed from measured data, such that the mean squared error (MSE) between the measured and estimated path loss is minimized.

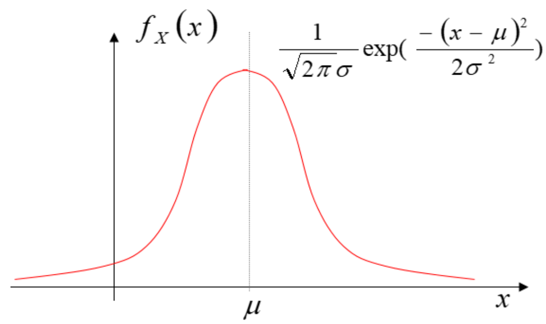
Note: $P_r(d)[dB]$ is also a r.v. with Gaussian (normal) distribution in dB.

(4) Log-normal Shadowing Model

Let X_σ be a Gaussian r. v., then the statistical distribution or probability distribution function (pdf) of X_σ is given by:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the average value of X, σ is standard deviation and σ^2 is the variance.



Thus

$$f_{PL(d)}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\overline{PL}(d))^2}{2\sigma^2}}$$

$$f_{p_r(d)}(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\overline{P_r}(d))^2}{2\sigma^2}}$$

Therefore, using the statistical information, we can determine the probability that the received power $P_r(d)$, or path loss $PL(d)$, will exceed (or fall below) a given level.

Mobile Communication Systems
Statistical Models – Shadowing

(4) Log-normal Shadowing Model

The probability that the received signal power (in dB) will exceed a certain value is given by

$$p(P_r(d) > \gamma) = \int_{\gamma_1}^{\infty} f_{P_r(d)} dx$$

$$p(P_r(d) > \gamma_1) = \int_{\gamma_1}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

No easy closed form solution so we will use the Q-function to calculate the answer. Let $y = \frac{x-\mu}{\sigma}$ then: $p(P_r(d) > \gamma_1) = \int_{\frac{\gamma_1-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dx = Q(\frac{\gamma_1-\mu}{\sigma})$

Q-function

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

Properties

- $Q(0) = 0.5$, $Q(-\infty) = 0$, $Q(\infty) = 1$
- $Q(x) = 0.5[1 - erf(\frac{x}{\sqrt{2}})] = 0.5 erfc(\frac{x}{\sqrt{2}})$
- $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$
- $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-y^2} dy$

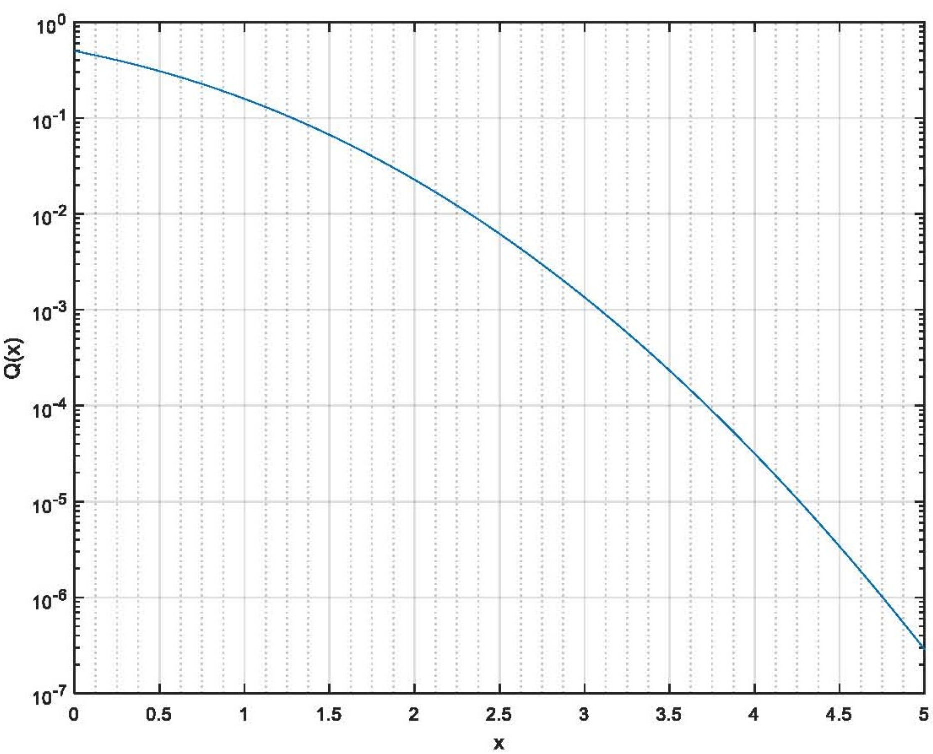
Similarly, the probability of the received signal power below γ_1 is given by:
$p(P_r(d) < \gamma_1) = 1 - Q(\frac{\gamma_1-\mu}{\sigma})$

Q-Function

Q(x)	0.00	1.00	2.00	3.00	4.00
0.00	0.5	0.158655254	0.022750132	0.001349898	3.17E-05
0.01	0.496010644	0.156247645	0.022215594	0.001306238	3.04E-05
0.02	0.492021686	0.15386423	0.021691694	0.001263873	2.91E-05
0.03	0.488033527	0.151505003	0.02117827	0.001222769	2.79E-05
0.04	0.484046563	0.14916995	0.020675163	0.001182891	2.67E-05
0.05	0.480061194	0.146859056	0.020182215	0.001144207	2.56E-05
0.06	0.476077817	0.1445723	0.01969927	0.001106685	2.45E-05
0.07	0.47209683	0.142309654	0.019226172	0.001070294	2.35E-05
0.08	0.468118628	0.14007109	0.018762766	0.001035003	2.25E-05
0.09	0.464143607	0.137856572	0.0183089	0.001000782	2.16E-05
0.10	0.460172163	0.135666061	0.017864421	0.000967603	2.07E-05
0.11	0.456204687	0.133499513	0.017429178	0.000935437	1.98E-05
0.12	0.452241574	0.131356881	0.017003023	0.000904255	1.89E-05
0.13	0.448283213	0.129238112	0.016585807	0.000874032	1.81E-05
0.14	0.444329995	0.127143151	0.016177383	0.000844739	1.74E-05
0.15	0.440382308	0.125071936	0.015777607	0.000816352	1.66E-05
0.16	0.436440537	0.123024403	0.015386335	0.000788846	1.59E-05
0.17	0.432505068	0.121000484	0.015003423	0.000762195	1.52E-05
0.18	0.428576284	0.119000107	0.014628731	0.000736375	1.46E-05
0.19	0.424654565	0.117023196	0.014262118	0.000711364	1.39E-05
0.20	0.420740291	0.11506967	0.013903448	0.000687138	1.33E-05
0.21	0.416833837	0.113139446	0.013552581	0.000663675	1.28E-05
0.22	0.412935577	0.111232437	0.013209384	0.000640953	1.22E-05
0.23	0.409045885	0.109348552	0.012873721	0.000618951	1.17E-05
0.24	0.405165128	0.107487697	0.012545461	0.000597648	1.12E-05
0.25	0.401293674	0.105649774	0.012224473	0.000577025	1.07E-05
0.26	0.397431887	0.103834681	0.011910625	0.000557061	1.02E-05
0.27	0.393580127	0.102042315	0.011603792	0.000537737	9.77E-06
0.28	0.389738752	0.100272568	0.011303844	0.000519035	9.34E-06
0.29	0.385908119	0.098525329	0.011010658	0.000500937	8.93E-06
0.30	0.382088578	0.096800485	0.01072411	0.000483424	8.54E-06
0.31	0.378280478	0.095097918	0.010444077	0.00046648	8.16E-06
0.32	0.374484165	0.093417509	0.010170439	0.000450087	7.80E-06
0.33	0.370699981	0.091759136	0.009903076	0.00043423	7.46E-06
0.34	0.366928264	0.090122672	0.00964187	0.000418892	7.12E-06
0.35	0.363169349	0.088507991	0.009386706	0.000404058	6.81E-06
0.36	0.359423567	0.086914962	0.009137468	0.000389712	6.50E-06
0.37	0.355691245	0.085343451	0.008894043	0.000375841	6.21E-06
0.38	0.351972708	0.083793322	0.008656319	0.000362429	5.93E-06
0.39	0.348268273	0.082264439	0.008424186	0.000349463	5.67E-06
0.40	0.344578258	0.080756659	0.008197536	0.000336929	5.41E-06
0.41	0.340902974	0.079269841	0.00797626	0.000324814	5.17E-06
0.42	0.337242727	0.077803841	0.007760254	0.000313106	4.94E-06
0.43	0.333597821	0.07635851	0.007549411	0.000301791	4.71E-06
0.44	0.329968554	0.0749337	0.007343631	0.000290857	4.50E-06
0.45	0.32635522	0.07352926	0.007142811	0.000280293	4.29E-06
0.46	0.32275811	0.072145037	0.006946851	0.000270088	4.10E-06
0.47	0.319177509	0.070780877	0.006755653	0.000260229	3.91E-06
0.48	0.315613697	0.069436623	0.006569119	0.000250707	3.73E-06
0.49	0.312066949	0.068112118	0.006387155	0.00024151	3.56E-06

Q-Function (cont.)

Q(x)	0.00	1.00	2.00	3.00	4.00
0.50	0.308537539	0.066807201	0.006209665	0.000232629	3.40E-06
0.51	0.305025731	0.065521712	0.006036558	0.000224053	3.24E-06
0.52	0.301531788	0.064255488	0.005867742	0.000215773	3.09E-06
0.53	0.298055965	0.063008364	0.005703126	0.00020778	2.95E-06
0.54	0.294598516	0.061780177	0.005542623	0.000200064	2.81E-06
0.55	0.291159687	0.060570758	0.005386146	0.000192616	2.68E-06
0.56	0.287739719	0.059379941	0.005233608	0.000185427	2.56E-06
0.57	0.284338849	0.058207556	0.005084926	0.000178491	2.44E-06
0.58	0.280957309	0.057053433	0.004940016	0.000171797	2.32E-06
0.59	0.277595325	0.055917403	0.004798797	0.000165339	2.22E-06
0.60	0.274253118	0.054799292	0.004661188	0.000159109	2.11E-06
0.61	0.270930904	0.053698928	0.004527111	0.000153099	2.01E-06
0.62	0.267628893	0.052616138	0.004396488	0.000147302	1.92E-06
0.63	0.264347292	0.051550748	0.004269243	0.000141711	1.83E-06
0.64	0.2610863	0.050502583	0.004145301	0.000136319	1.74E-06
0.65	0.257846111	0.049471468	0.004024589	0.00013112	1.66E-06
0.66	0.254626915	0.048457226	0.003907033	0.000126108	1.58E-06
0.67	0.251428895	0.047459682	0.003792562	0.000121275	1.51E-06
0.68	0.24825223	0.046478658	0.003681108	0.000116617	1.43E-06
0.69	0.245097094	0.045513977	0.003572601	0.000112127	1.37E-06
0.70	0.241963652	0.044565463	0.003466974	0.0001078	1.30E-06
0.71	0.238852068	0.043632937	0.00336416	0.00010363	1.24E-06
0.72	0.235762498	0.042716221	0.003264096	9.96E-05	1.18E-06
0.73	0.232695092	0.041815138	0.003166716	9.57E-05	1.12E-06
0.74	0.229649997	0.040929509	0.003071959	9.20E-05	1.07E-06
0.75	0.226627352	0.040059157	0.002979763	8.84E-05	1.02E-06
0.76	0.223627292	0.039203903	0.002890068	8.50E-05	9.68E-07
0.77	0.220649946	0.03836357	0.002802815	8.16E-05	9.21E-07
0.78	0.217695438	0.03753798	0.002717945	7.84E-05	8.76E-07
0.79	0.214763884	0.036726956	0.002635402	7.53E-05	8.34E-07
0.80	0.211855399	0.035930319	0.00255513	7.23E-05	7.93E-07
0.81	0.208970088	0.035147894	0.002477075	6.95E-05	7.55E-07
0.82	0.206108054	0.034379502	0.002401182	6.67E-05	7.18E-07
0.83	0.203269392	0.033624969	0.0023274	6.41E-05	6.83E-07
0.84	0.200454193	0.032884119	0.002255677	6.15E-05	6.49E-07
0.85	0.197662543	0.032156775	0.002185961	5.91E-05	6.17E-07
0.86	0.194894521	0.031442763	0.002118205	5.67E-05	5.87E-07
0.87	0.192150202	0.030741909	0.002052359	5.44E-05	5.58E-07
0.88	0.189429655	0.030054039	0.001988376	5.22E-05	5.30E-07
0.89	0.186732943	0.02937898	0.001926209	5.01E-05	5.04E-07
0.90	0.184060125	0.02871656	0.001865813	4.81E-05	4.79E-07
0.91	0.181411255	0.028066607	0.001807144	4.61E-05	4.55E-07
0.92	0.17878638	0.02742895	0.001750157	4.43E-05	4.33E-07
0.93	0.176185542	0.026803419	0.00169481	4.25E-05	4.11E-07
0.94	0.17360878	0.026189845	0.001641061	4.07E-05	3.91E-07
0.95	0.171056126	0.02558806	0.00158887	3.91E-05	3.71E-07
0.96	0.168527607	0.024997895	0.001538195	3.75E-05	3.52E-07
0.97	0.166023246	0.024419185	0.001488999	3.59E-05	3.35E-07
0.98	0.163543059	0.023851764	0.001441242	3.45E-05	3.18E-07
0.99	0.16108706	0.023295468	0.001394887	3.30E-05	3.02E-07



Outage Probability

Outage probability

Due to shadowing, the received power at any given distance d from the BS may fall below the minimum usable power required by the mobile station, P_{min}. We define the outage probability $P_{out}(P_{min}, d)$ as:

$P_{out}(P_{min}, d) = p(P_r(d) < P_{min}) = 1 - Q(\frac{P_{min} - \overline{P_r}(d)}{\sigma})$

Where $\overline{P_r}(d) = \overline{P_r}(d_0) + 10n\log_{10}(\frac{d_0}{d}) = P_t + 20\log_{10}(\frac{\lambda}{4\pi d_0}) + 10n\log_{10}(\frac{d_0}{d})$

Example 3:

Find the outage probability at 150m from a BS for a channel based on the combined path loss and shadowing models, assuming transmitted power of $P_t = 10$mW, minimum power requirement of $P_{min} = -110.5$dBm, $\overline{P_r}(d_0)[dB] = -31.54$ dB, $d_0 = 1$m, $\sigma = 3.65$, and $n = 3.71$.

Solution

$P_{out} = 0.0121$

$P_{out} = 1\%$ is a typical target in wireless system designs.

Outage Probability

Example 4:

A mobile receiver in a cellular system detects 1 mW signal at a distance $d_0 = 1$m from a serving base station. Assuming hexagonal cell geometry, and cell radius $R = 470$m in a 7-cell reuse system, compute:

1. The average received signal power at the edge of the cell (given that the path loss exponent for the environment is $n = 3$).
2. The probability that a call at the edge of the cell is not lost (assuming log-normal shadowing with variance $\sigma^2 = 16$), given that the minimal acceptable signal level to ensure a call is not lost is $\gamma = -108$ dB

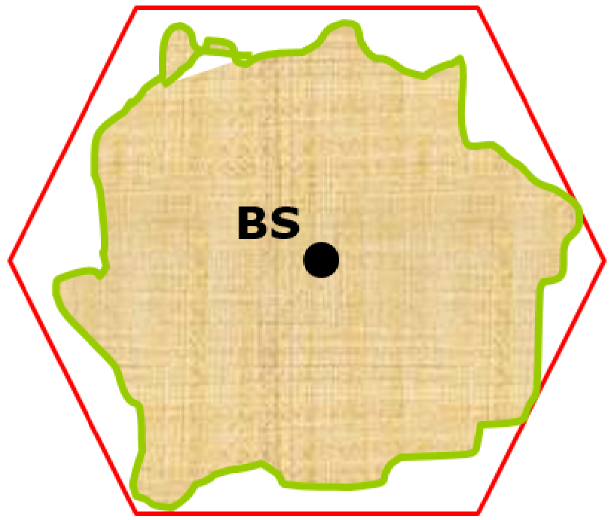
Solution

1. $P_r(d) = 10\log_{10} P_r(d_0)[\frac{R}{d_0}]^{-n} = -110.17$ dB
2. $p[P_r(R) > \gamma = -108dB] = Q[(\gamma - P_r(R))/\sigma] = Q(0.5425)$
   Hence p[call not lost] = $p[P_r(R) > \gamma = -108dB] = 0.3$

Determination of % Coverage Area

Useful Cell coverage area:

- It is clear that due to random effects of shadowing, some locations within a coverage area will be below a particular desired received signal threshold.
- Figure here illustrates the challenges that shadowing poses in cellular system design.
- Effective (useful) coverage area now looks like amoeba-like shape, and worse still this shape is constantly changing due to the random nature of shadowing!.



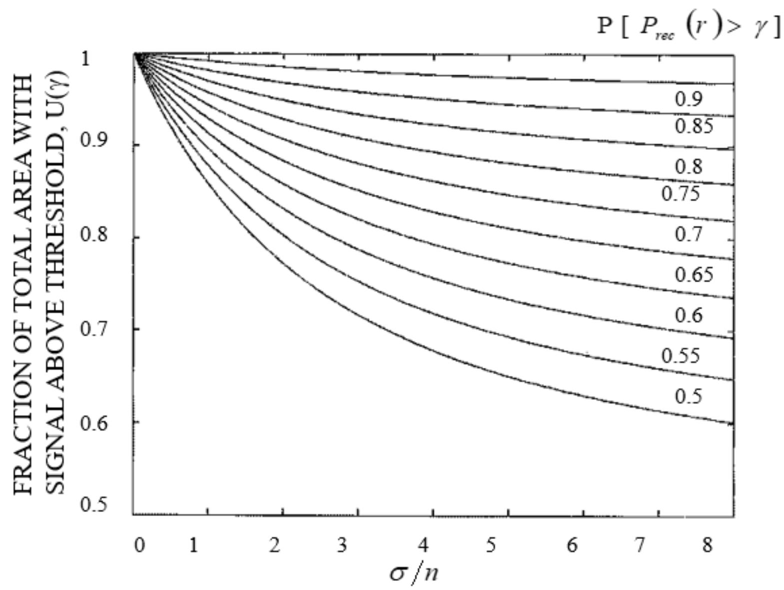
Determination of % Coverage Area

Useful Cell coverage area:

- For a coverage area of radius R from a BS, let γ be the desired usable received signal threshold.
- It can be shown that the % of useful service area (i. e., the % of area with the average received signal $\geq \gamma$), given a known likelihood of coverage at the cell boundary is: $U(\gamma) = \frac{1}{2}\left(1 - erf(a) + e^{\frac{1-2ab}{b^2}}\left[1 - erf(\frac{1-ab}{b})\right]\right)$ where
  $a = \frac{\gamma - \overline{P_r}(R)}{\sqrt{2}\sigma} = \frac{\gamma - P_t + \overline{PL}(d_0) + 10n\log(\frac{R}{d_0})}{\sqrt{2}\sigma}$ and $b = \frac{10n\log_{10} e}{\sqrt{2}\sigma}$
- By choosing the signal level such that $\overline{P_r}(R) = \gamma$ i.e. $a = 0$, we have:
  $U(\gamma) = \frac{1}{2}\left(1 + e^{\frac{1}{b^2}}\left[1 - erf(\frac{1}{b})\right]\right)$ (entire cell is usefully covered ?)
- In terms of the Q-function, we have respectively for these two cases:
  $U(\gamma) = Q(\hat{a}) + e^{\frac{2-2\hat{a}\hat{b}}{\hat{b}^2}} Q(\frac{2-\hat{a}\hat{b}}{\hat{b}})$ where $\hat{a} = \frac{\gamma - \overline{P_r}(R)}{\sigma}$ and $\hat{b} = \frac{10n\log_{10} e}{\sigma}$
  $U(\gamma) = \frac{1}{2} + e^{\frac{2}{\hat{b}^2}} Q(\frac{2}{\hat{b}})$

Determination of % Coverage Area

U(γ) can be evaluated for a large number of values of σ and n as shown below



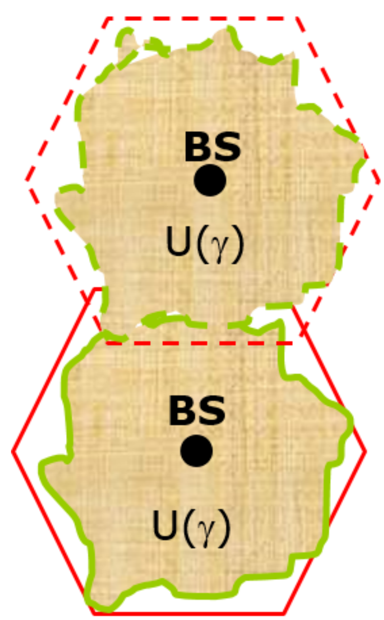
Determination of % Coverage Area

Example 5:

1. If $n = 4$ and $\sigma = 8$ dB, and if 75% boundary coverage is desired (i.e., 75% of time the signal is to exceed the threshold, γ, at the boundary), then the area coverage is 90% ($U(\gamma) = 0.9$).
2. If $n = 2$ and $\sigma = 8$ dB ($\sigma/n = 4$), a 75% boundary coverage provides 86% area coverage
3. If $n = 3$ and $\sigma = 9$ dB ($\sigma/n = 3$), then 50% boundary coverage provides 71% area coverage

Solution

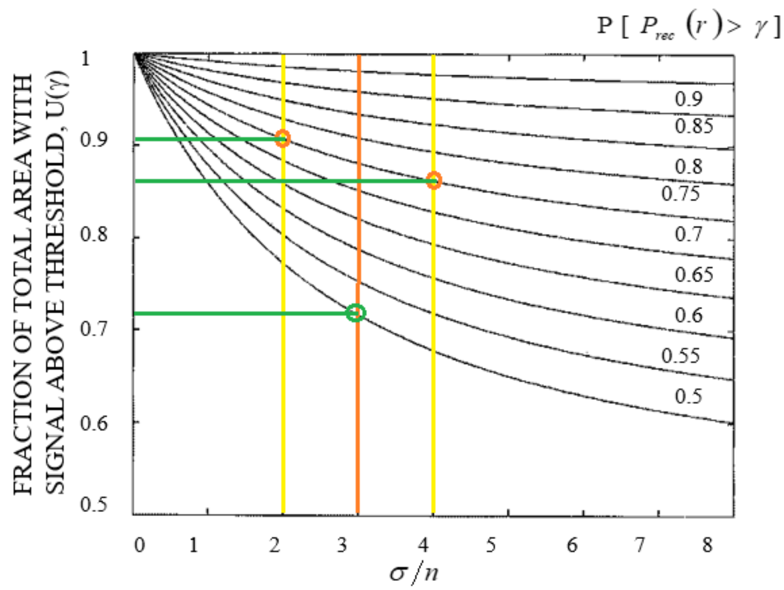
See previous slide for chart.



Cell coverage overlap at boundary to avoid gaps

Determination of % Coverage Area

U(γ) can be evaluated for a large number of values of σ and n as shown below



Determination of % Coverage Area

Example 6:

1. Find the coverage area for a cell with the combined path loss and shadowing model of Example 3.4, a cell radius of 600m, a BS transmit power of $Pt = 100$ mW which equal 20dBm, and a minimum received power requirement of $P_{min} = -110$ dBm.
2. Repeat part (a) for the case $P_{min} = -120$ dBm.

Solution

1. $P_r(R) = P_r(d_0) - 10n\log_{10}(600) = -114.6$ dBm, $\hat{(a)} = \frac{-110+114.6}{3.65} = 1.26$, $\hat{b} = \frac{37.1(0.434)}{3.65} = 4.41$. and $U(\gamma) = Q(1.26) + e^{\frac{2-2(1.26)(4.41)}{4.41^2}} Q(\frac{2-(1.26)(4.41)}{4.41})$
   Thus, $U(\gamma) = 0.59$
2. $U(\gamma) = 0.988$

Example 7:

Four received power measurements were taken at 100m, 200m, 1km, and 3km from a TX. These measured values are shown in table below. It is assumed that the PL for these measurements follows the log-normal shadowing model, where $d_0 = 100$ m:

1. Find the minimum mean square error (MMSE) estimate for PLE n.
2. Calculate the standard deviation about the mean value.
3. Estimate the received power at d=2km using the resulting model.
4. Predict the prob that received signal level at 2km will be greater than -60 dBm.
5. Predict the % of area within a 2km radius cell that receives signals greater than –60dBm, given the result in part (d).

d	$P_r(d)$
100 m	0 dBm
200 m	-20 dBm
1000 m	-35 dBm
3000 m	-70 dBm

Solution

From $\hat{P}_r(d)[dB] = P_r(d_0)[dB] + 10n\log_{10}(\frac{d_0}{d})$

i	d_i	$P_r(d_i)$	$\hat{P}_r(d_i)$
1	100 m	0 dBm	$0 + 10n\log_{10}(1) = 0$
2	200 m	-20 dBm	$0 + 10n\log_{10}(1/2) = -3n$
3	1000 m	-35 dBm	$0 + 10n\log_{10}(1/10) = -10n$
4	3000 m	-70 dBm	$0 + 10n\log_{10}(1/30) = -14.7n$

1. The mean squred error (MSE) is given by:

$$MSE = J(n) = \sum_{i=1}^{N}\left(P_r(d_i)[dB] - \hat{P}_r(d_i)[dB]\right)^2$$

$$J(n) = (0-0)^2 + (-20+3n)^2 + (-35+10n)^2 + (-70+14.7n)^2$$

To minimize MSE, the derivative with respect to n should be zero: $\frac{\partial J(n)}{\partial n} = 0$

$$2(-20+3n)(3) + 2(-35+10n)(10) + 2(-70+14.7n)(14.7) = 0$$

Therefore, $n = 4.4$

Solution

2. The standared deviation σ is calculated at the minimum mean sqared value at $n = 4.4$. Therefore, $\sigma = \sqrt{\frac{1}{4}\sum_{i=1}^{4} J(n)|_{n=4.4}} = 6.17$ dB.
3. $P_r(2km)[dB] = P_r(d_0)[dB] + 10n\log_{10}(\frac{d_0}{d})|_{d=2km} = -57.25$ dB.
4. $p(P_r(2km) > -60dB) = Q(\frac{-60+57.25}{6.17}) = 0.69$
5. $\frac{\sigma}{n} = \frac{6.17}{4.4} = 1.4$ then $U(\gamma) = 0.85$.

Homework 3

The table below is a set of measurements conducted in the 38 GHz band of the millimeter wave spectrum in New York City. Find a path loss (PL) exponent, n and the standard deviation σ that fit the measured data.

Distance from BS to MS	Measured Path loss at 38 GHz in dB
5 m	0.2898
10 m	6.738
20 m	13.33
30 m	17.2
40 m	19.95
50 m	22.03
100 m	28.72
150 m	32.4
200 m	35.35
250 m	37.49
300 m	39.34
350 m	40.72